Trigonometry (5/5): Inverse Trigonometric Functions

Introduction to Engineering Mathematics

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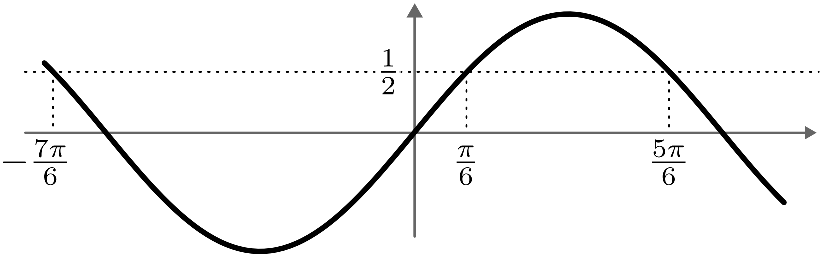
Overview

1. Definition of the inverse trigonometric functions
2. Examples

Inverting the sine function

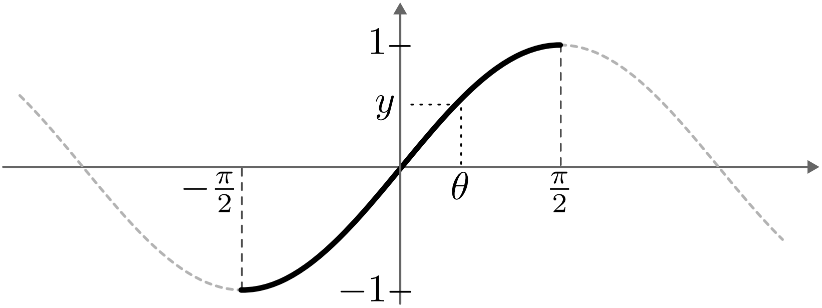
- The sine function turns angles into sine values.
- The **inverse sine** turns sine values back into angles.
- Notation: $\sin^{-1}(x)$, $\arcsin x$

Problem: Many values in the range correspond to the same angle!



Restricting the domain

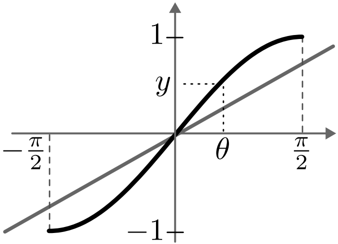
Solution: restrict the domain of the sine function so that there is exactly one angle corresponding to each value.



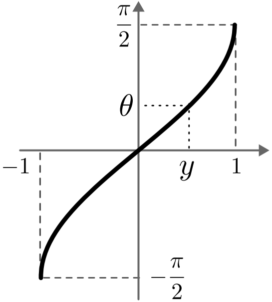
This gives us a meaningful way to define the inverse sine.

The inverse sine function

- Domain: $[-1, 1]$, range: $[-\pi/2, \pi/2]$
- Cancellation properties:
  - $\sin(\arcsin(y)) = y$ for all $y \in [-1, 1]$
  - $\arcsin(\sin(\theta)) = \theta$ for all $\theta \in [-\pi/2, \pi/2]$.



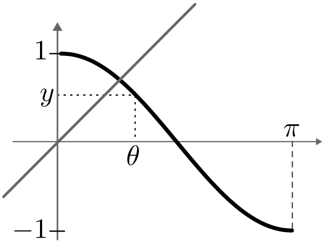
$y = \sin\theta$



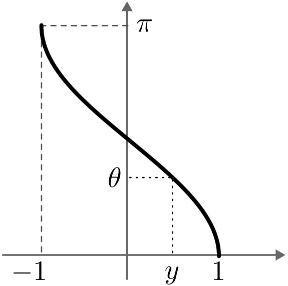
$\theta = \arcsin y$

The inverse cosine function

- Domain: $[-1, 1]$, range: $[0, \pi]$
- Cancellation properties:
  - $\cos(\arccos(y)) = y$ for all $y \in [-1, 1]$
  - $\arccos(\cos(\theta)) = \theta$ for all $\theta \in [0, \pi]$



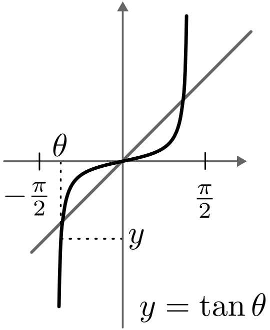
$y = \cos\theta$

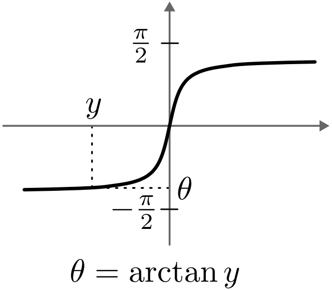


$\theta = \arccos y$

The inverse tangent function

- Domain: all of $\mathbb{R}$, range: $[-\pi/2, \pi/2]$
- Cancellation properties:
  - $\tan(\arctan(y)) = y$ for all $y \in \mathbb{R}$
  - $\arctan(\tan\theta) = \theta$ for all $\theta \in [-\pi/2, \pi/2]$





Notation (warning)

Don't confuse

$$\sin^k \theta = (\sin \theta)^k, \quad \text{for } k \geq 0$$

with

$$\sin^{-1} x = \arcsin x.$$

Example

Simplify the following expression: $\tan\left(\sin^{-1}\frac{2}{3}\right)$.

Example

Show that $\arccos x = \frac{\pi}{2} - \arcsin x$.