Trigonometry (1/5): Introduction and Overview

Introduction to Engineering Mathematics

Prof. Joris Vankerschaver

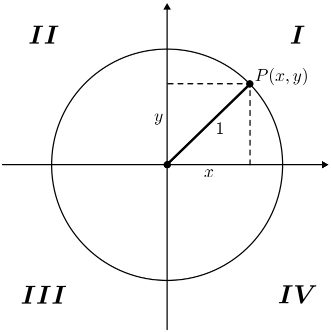
Contents

1. Angles and points on the unit circle
2. Trigonometric functions as coordinates
3. Basic trigonometric identities
4. Graphs of trigonometric functions

Angles and points on the unit circle

The unit circle

- The circle of radius 1 in the xy-plane, centered on the origin.
- Equation: $x^2 + y^2 = 1$
- Four quadrants: I, II, III, IV

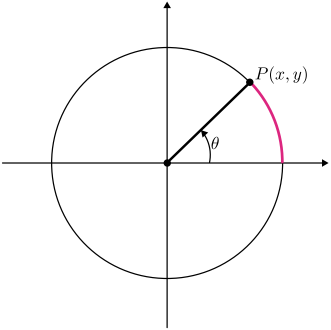


Example

If $P(\sqrt{3}/2, y)$ is a point on the unit circle, find the value of y.

Angles and points on the unit circle

- Each point $P(x, y)$ defines an angle θ measured from the positive x-axis in counterclockwise direction.
- Angles measured in degrees or radians.
    - Value of θ in radians: length of arc subtended by θ (length of the red segment)

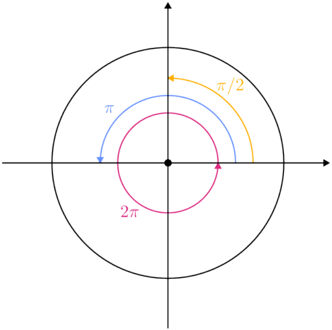


Converting between angles and radians

General formula to convert between degrees and radians:

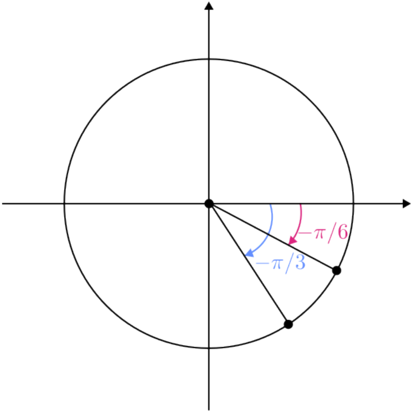
$$\text{degrees} \underset{\times\frac{180}{\pi}}{\overset{\times\frac{\pi}{180}}{\rightleftarrows}} \text{radians}$$

	Degrees	Radians
Full circle	360°	2π
Half circle	180°	π
Quarter circle	90°	$\pi/2$



Negative angles

Measured from the positive x-axis, in clockwise direction.

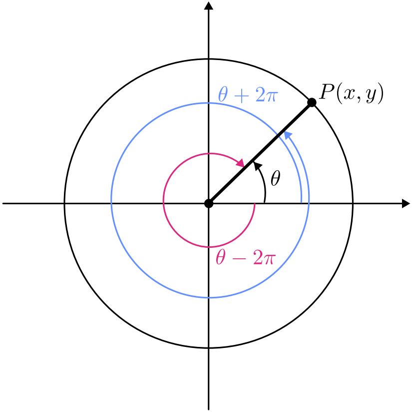


Adding 2π to an angle

- Point P is determined by the angle θ.
- P stays same when adding $\pm 2\pi$ to θ.

$\Rightarrow$ All angles $\theta + 2k\pi$ with $k \in \mathbb{Z}$ give the same point P.

Principal angle: θ such that $-\pi < \theta \leq \pi$.

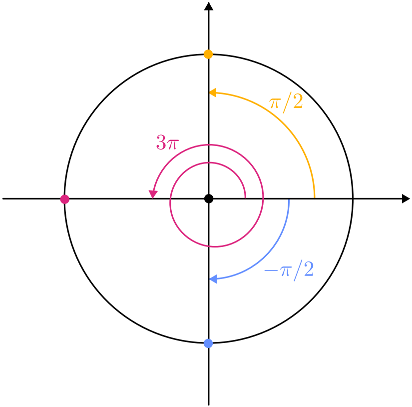


Trigonometric functions as coordinates

Finding the coordinates of a point

Given an angle θ, find the coordinates of $P(x, y)$.

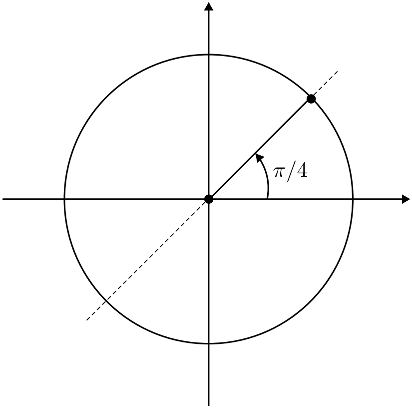
1. $\theta = \pi/2$
2. $\theta = 3\pi$
3. $\theta = -\pi/2$



Finding the coordinates of a point

Slightly more involved case:

4. $\theta = \pi/4$



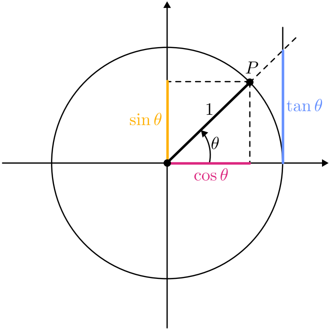
Important angles

Angle	x-coordinate	y-coordinate
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1
π	-1	0
2π	1	0

Trigonometric functions as coordinates

Let θ be an angle with point $P(x, y)$.

Name	Notation	Definition
Cosine	$\cos\theta$	x
Sine	$\sin\theta$	y
Tangent	$\tan\theta$	$\frac{\sin\theta}{\cos\theta}$
Cotangent	$\cot\theta$	$\frac{\cos\theta}{\sin\theta}$
Cosecant	$\csc\theta$	$\frac{1}{\sin\theta}$
Secant	$\sec\theta$	$\frac{1}{\cos\theta}$



Example

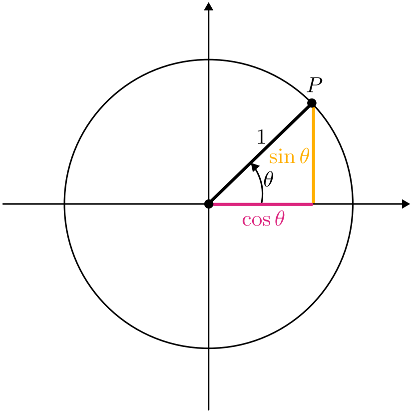
Given that $\theta = \dfrac{\pi}{6}$, find the values of all 6 trigonometric functions.

Basic trigonometric identities

Fundamental identity

- $P(x, y)$ is on the unit circle: $x^2 + y^2 = 1$
- Put $x = \cos\theta$ and $y = \sin\theta$ to obtain the **fundamental identity**:

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$



Aside: notation

Be very careful when you see $\sin^k \theta$.

- Positive exponent (**power**):

$$\sin^k \theta = (\sin \theta)^k.$$

- Negative exponent -1 (**inverse function**):

$$\sin^{-1} y = \arcsin y.$$

Fundamental identity: consequences

$$1 + \tan^2\theta = \sec^2\theta \qquad\qquad 1 + \cot^2\theta = \csc^2\theta$$

Example

Suppose $\cos\theta = -\dfrac{4}{5}$ and θ is in quadrant III. Find $\sin\theta$ and $\tan\theta$.

Periodicity of sine and cosine

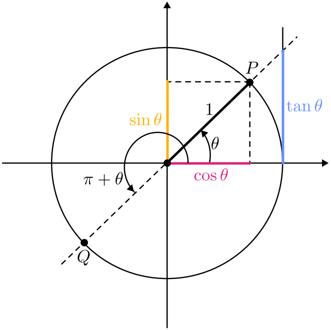
- Sine and cosine are 2π-periodic:

$$\sin(\theta \pm 2\pi) = \sin\theta$$
$$\cos(\theta \pm 2\pi) = \cos\theta$$

- The tangent is π-periodic:

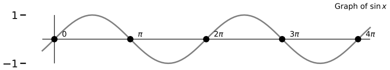
$$\tan(\theta \pm \pi) = \tan\theta$$

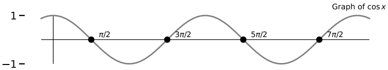
Example: Compute $\tan\left(\dfrac{8093\pi}{4}\right)$



Graphs of trigonometric functions

Graphs of sine/cosine

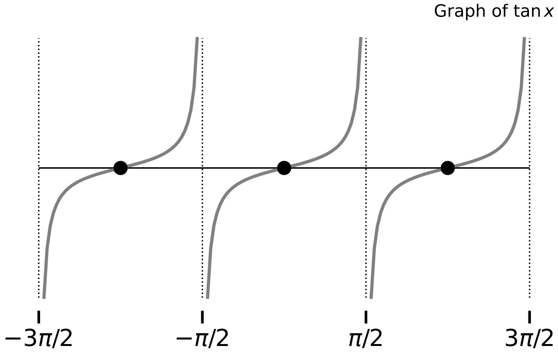




The sine and cosine:

- Are defined for every real number.
- Oscillate between -1 and $+1$.
- Repeat themselves every 2π radians (**fundamental period**).

Graph of the tangent function



The tangent function:

- Is defined for every real number, **except multiples of** $\pi/2$.
- Can take on arbitrary values.
- Repeats itself every π radians (fundamental period).

Example

Find the fundamental period of $\sin(2x)$.